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DECENTRALIZED CONTROL AND DECENTRALIZED ADAPTIVE CONTROL

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14. ABSTRACT

As systems become complex with many interconnected subsystems, decentralized control becomes essential. When certain parameters of the system are unknown, and/or when subsystems are not aware of the signals from other subsystems that affect their behavior, we need decentralized adaptive control. The report deals with questions that arise while analyzing the stability and performance of decentralized adaptive control systems. The project produced three specific results: 1. Interconnected dynamical systems can be stable even when there is no communication between subsystems, provided all subsystems have common knowledge of the goals of the other subsystems. 2. Even though stability can be achieved without communication, the latter is necessary to satisfy performance requirements. To keep communication costs to a minimum, partial communication has to be used. This gives rise to stability problems which were resolved. 3. The problem as to when a subsystem in an interconnected-system communicates with another is an important one and needs to be investigated further. Simulation results have clearly shown that significant improvement in the performance of the overall system can be achieved by subsystems communicating only over critical intervals of time.

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Decentralized decision making, control theory, adaptive control, biological comparison, communication networks

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Table of Contents

1. Introduction and Overview

1.1 Introduction

Decentralized decision making is ubiquitous in both man made (artificial) systems and biological (natural) systems. As systems become more complex, with many interconnected subsystems, decentralized control becomes essential. Examples of such systems include Economic Systems, Communication Networks, Transportation Systems, Power Systems and Unmanned Air Vehicles (UAV).

In economic markets with millions of buyers and sellers, the traders do not make decisions collectively. Instead, the price of any commodity or stock is the only commonly available variable, and the collective decisions of all individuals decides the price. In communication networks messages from multiple source nodes are routed to multiple destination nodes. To avoid congestion and improve the overall performance of the network, a better understanding of the effects of decentralization is needed. In transportation systems, where millions of hours are being wasted every day, it is now recognized that the solution of decentralized control problems will require novel concepts in the future. Events like the blackout in New York serve to remind us that our understanding of the stability of interconnected power systems is still rather limited. At the present time there is a great deal of interest in the design of unmanned air vehicles in high-risk, time-critical, and high-pay-off situations. Such systems must be capable of collecting, correlating, and sharing available data for collision avoidance, contingency planning, and to perform cooperative tasks. All these require a much deeper understanding of decentralized control systems than we currently possess.

1.2 Biological Motivations

Biological systems cope easily and effectively with changes in their environments. Unlike artificial systems there is a seamless integration in them of the activities of the many agents involved. Through evolution, ants, wasps, bees, birds, and fishes etc have developed extremely efficient methods for finding food, dividing labor, feeding the colony, and responding to external challenges. All these reveal that nature, has come up with novel solutions to complex problems which need to be better understood. More specifically, question such as relation between simple individual behavior and complex collective behavior, the principles that govern the stability of the equilibrium state of the community, the process by which cooperation arises among a multiplicity of agents, must all be investigated in detail analytically.

1.3 Decentralization and Uncertainty

There is always uncertainty that is associated with decentralization. However, uncertainty (due to parameter variations, poor modeling, and external disturbances) can also exist without decentralization. The model used to represent a specific decentralized situation depends upon the degree to which decentralization and uncertainty are important in it. The broad field of adaptive control has developed over the past forty years as an effective control methodology for dealing with uncertainty. This accounts for the title of the project, which includes both decentralized control as well a decentralized adaptive control.

1.4 Objective of the Proposed Research

From the history of control theory it is well known that efficient design principles could be developed for linear time-invariant (LTI) systems and (linear) adaptive systems only when their stability properties were well understood. The PI adopts the same viewpoint towards the decentralized control of complex interconnected systems, and believes that their stability properties must be understood before their design can be attempted with confidence.

When a number of subsystems are interconnected, and each subsystem has parametric uncertainty as well as disturbances (from unknown signals) from other subsystems, the overall system may become unstable. Determining the strategies by which the different subsystems must compute their control input to ensure stability is consequently an important question. It is only after conditions for stability are well understood can the designer focus his/her attention on improving the performance of individual subsystems. The first objective of this project was to study in detail the stability of interconnected systems with no communication between the subsystems and later to extend the results to the case where communication between subsystems is permitted. Following this, the objective of the project was to determine methods for improving the performance of the overall system within a stability framework.

1.5 Statement of the Problems

N subsystems $\Sigma_1, \Sigma_2, \ldots, \Sigma_N$ are interconnected to form a network of systems (e.g. power systems). Each subsystem is a dynamical system and has unknown or time-varying parameters. Each subsystem Σ_i has also a well defined objective (i.e. that its output must follow a specified function of time y_{di}). The fact that the objective has to be accomplished in the presence of uncertainty makes each subsystem adaptive. In addition to local uncertainty it is also assumed that each subsystem Σ_i is affected by signals from one or more of the other subsystems Σ_j ($j \neq i$) and that these signals are not known to Σ_i. If Σ_i were to choose its control input without knowledge of these signals from the other subsystems, its output will not be able to follow the desired output. More to the point, since this applies to all the other subsystems also, the overall system may become unstable. Hence, two questions need to be answered in the strictly decentralized case.

(i) If the overall system is to be stable what prior information concerning the other subsystems will be needed to control each subsystem in the network?

(ii) Is it theoretically possible to control each subsystem so that all the subsystems achieve their objective of following their desired outputs asymptotically with zero error?

In the late 1990s researchers found partial answers to question (i). In January 2000 the PI made a conjecture concerning Problem (ii). Qualitatively, this conjecture states that all the subsystems could achieve their objectives if each subsystem is aware of the goals of the other subsystems. In other words, the desired outputs of all N subsystems should be common knowledge. After two years of theoretical investigation, the conjecture was verified analytically for a simple class of linear systems with linear interactions. The research carried out in this contract was strongly influenced by the above result for two very different reasons that are given below

(i) The conjecture was verified for only simple systems. For use in practical engineering problems, it would be desirable to determine the most general class of systems for which it is valid.

(ii) As mentioned in the introduction, there are a large number of natural systems in existence where complex overall system behavior results from simple interaction of individual members. The conjecture raises the possibility that similar simple explanations can be found for the behavior observed in those systems as well.

In this project, the following three sets of problems were posed for further investigation:

(i) The first deals with the strictly decentralized control system discussed thus far. For all the systems investigated simulation studies would have to be carried out to determine how they would perform under real world conditions.

(ii) In the second set of problems the restriction concerning communication between subsystems should be removed and stability issues should be investigated. Once again, for cases where stability can be assured, methods for improving performance within a stability framework should be undertaken.

(iii) While achieving the objectives of individual subsystems (and hence of the collective) is important, the cost of communication should also be taken into account while proposing control strategies. In particular, methods must be investigated to determine the minimum amount of communication that will be needed to keep the output errors of the N subsystems within specified limits.

1.6 Contributions of the Project

Work carried out for the project has conclusively shown that

(a) The conjecture is applicable to a wide class of dynamical systems. This result alone has extended significantly the class of theoretical problems to which adaptive control can be applied.

(b) Simulation studies on strictly decentralized control systems revealed that such systems are very sensitive to initial parameter and output errors. Hence, it was concluded that for designing practically viable interconnected systems, communication between subsystems should be permitted.

Partial communication between subsystems raises difficult stability questions described later in this report. Another important contribution of the project was the proof of stability of interconnected systems with partial communication.

(c) Since the stability proofs given in (b) demonstrate that the overall system can be made stable no matter when the different subsystems communicate with each other, they naturally lead to the question as to when, what, and with whom each subsystem should communicate to achieve satisfactory performance while minimizing communication costs. Heuristic rules for determining this demonstrated that significant improvement in performance (comparable to that resulting from total communication) could be achieved by communicating at critical instants of time.

2. Work Accomplished

2.1 Strictly Decentralized Adaptive Control

The problem of strictly decentralized adaptive control may be stated as follows: Every set of $(N - 1)$ subsystems affect the input to the Nth subsystem, in a network composed of N subsystems. The inputs to the Nth system are output variables of the other subsystems, and are generally unknown. The question that is to be addressed is the strategy that each subsystem should take so that all the signals in the system are bounded, and if possible the outputs of the subsystems follow their desired outputs exactly as $t \to \infty$.

In the work that was carried out before 2002 most of the workers treated the outputs of the other subsystems as “disturbances” and attempted to derive conditions under which the overall system would have bounded outputs. In 2002 the PI and his graduate student showed that if the various controllers used desired values instead of the true values of the signals, all errors would eventually converge to zero. However, this was demonstrated for two linear systems all of whose relevant variables (state variables) are accessible. Further, both systems are linear with unknown parameters and interconnections between them.

The work done during the first part of the project aimed at demonstrating that this result would also carry over to a large class of systems. This was accomplished early in 2005.

The first extension was to interconnected systems when the interconnections are nonlinear. However, to make the problems mathematically tractable, several assumptions had to be made about the nonlinear functions. Finding conditions for stability involved determining non-quadratic Lyapunov functions, an area in which relatively little work has been done. The above positive result prompted the PI and his graduate students to investigate more general interconnected systems.

In the next stage, attempts were made to extend subsystems where each subsystem has access only to its output and not all the relevant (state) variables.

For special classes of such systems it was shown that relatively simple adaptive laws could be derived using the available information. However, the problem becomes quite involved mathematically. Simulation results were also carried out to ensure that all the error signals did converge to zero.

Attempts to extend the results to more general classes of systems (with all the state variables not accessible) required estimates of these variables. This consequently involved considerably more mathematical machinery involving both observers and backstepping. However, the problem was finally resolved.

As a result of all the above efforts it was concluded that the idea that desired values (known a priori) in place of actual values (measured on-line) could still result in asymptotic stability, was a general adaptive principle applicable to a large class of dynamical systems.

As stated earlier, simulation studies on strictly decentralized control systems using the approach was carried out concurrently. Even though the theory is very attractive, the practical application of the principle was disappointing. For relatively small parametric uncertainly and large errors in initial conditions of some variables, the errors were found to be quite large. It was there fore decided that control problems where subsystems can communicate with each other should be investigated next.

2.2 Decentralized Adaptive Control with Communication

The Principal difficulty in proving the stability of the systems discussed in the previous section lies in the fact that the relevant signals in the other subsystems that affect a specific subsystem are not accessible to it. This accounted for using their desired values in the adaptive laws. If however, the subsystems could communicate with each other, they could convey the information needed to make the system stable (i.e. the values of the corresponding variables).

If communication between subsystems is permitted, an extreme case would correspond to every subsystem communicating with ever other subsystem at every instant! In this case the system is no longer decentralized, and the results will be identical to that obtained with a centralized controller.

The problem therefore is to determine when any subsystem should communicate with another. This in turn led to the problem of stability when every subsystem receives information from others, not at all instants but only intermittently. Assuming that a subsystem receives information in such a fashion what strategy should it adopt? When the true values of the signals are known, simple stable adaptive laws for generating the input can be easily determined. In the absence of such information the subsystem has to use prior information and that could involve the desired values of the same signals. With intermittent communication, it is clear that the subsystem has to switch between two different strategies – one using the true value of the “disturbing” signals and the other using their desired values.

A substantial amount of work has been done in the past fifteen years on the stability of switching systems. It has been shown that if a common Lyapunov function exists for two dynamical systems, then a system switching between them at arbitrary instants is globally stable. In the context of interconnected systems, where all the subsystems switch between two strategies, it becomes necessary to find a common global Lyapunov function. This was accomplished by the PI and his colleagues, and is scheduled to appear as a paper in a special issue of the Institute of Electrical Engineers of the United Kingdom in September, 2006. (see Appendix)

The results described above imply that the existence of a common Lyapunov Function decouple completely the problem of stability and performance. Since stability is assured, every subsystem is free to choose when to communicate with the other subsystems to improve performance.

2.3 Communication and Optimization

In the last stage of the project, conditions for improving performance were investigated by determining when communications should take place. Extensive simulation results were carried out, some of which are described in Section 3. As seen from these studies, communication at some critical moments can substantially improve performance. So determining when communication should take place was posed as an optimization problem. The performance criterion used in the optimization procedure depends both on the cost of communication as well as the performance of the overall system as represented by the output errors. Work in this area was in progress when the project was terminated.

3. Experimental Results

Extensive simulation studies were carried out both on continuous-time systems and discrete time systems to verify the theoretical results described in Section 2. While there are essential differences between the theory of adaptive systems in continuous and discrete time, the experimental results observed in simulation studies in the two cases were quite similar.

The experiments were carried out on two classes of systems. The first included only two interconnected systems so that the effect of different parameters could be critically investigated.

In the second class, the behavior of six interconnected subsystems was studied. In the latter case, thirty interconnections are possible making the system both complex and realistic

3.1 Simulation Studies: (Two Interconnected Systems)

Even without the interconnection, the two individual subsystems can have unknown parameters and hence adaptive control had to be used in both cases. The two systems were chosen to be linear with linear interconnections, and both systems were unstable.

The questions of interest are

> (i) How do the individual subsystems stabilize themselves when no interconnection is present?
>
> (ii) How do the interconnections affect the behavior of the two systems when (a) there is no communication, (b) partial communication, and (c) complete communication between them?

In the case when interconnection is present, each subsystem is unaware of the signals of theother system that are affecting its output.

Simulation studies carried out with the two adaptive subsystems decoupled were carried out with sinusoidal reference inputs. While the output errors converged to zero as expected, the overshoots of the corresponding signals were 5.12 and 2.80 respectively. These were used as benchmark values while evaluating the performance of the interconnected systems.

Several simulations were carried out assuming that the two subsystems ignore the interconnection and treat the signals from the other subsystems as disturbances. Since, this corresponds to robust adaptive control, none of the signals converged to their desired values. Further, in some cases the overall system became unstable. This, in turn, indicates that in practical systems the external signal cannot be treated as “disturbances” and that active control action has to be taken to compensate for them.

In the next set of simulation studies, the desired (sinusoidal) outputs of the two systems were assumed to be common knowledge, Hence, each subsystem uses the desired (rather than the actual) output of the other subsystem in its computation of the control input. In view of this, the overall system became stable. However, the overshoots in the transient case were substantially larger as expected.

In the following simulation studies communication played a central role in the control problem. The effect of both complete communication and partial communication on overall performance were studied. As stated in Section 2, the degree of communication can be varied by changing a threshold parameter T i.e. the system Σ_i measures its own output y_i, and the error $y_{di} - y_i$ and communicates the value y_i to the other subsystem when $|y_{di} - y_i| \geq T$. $T = 0$ implies communication at every instant while $T = \infty$ implies no communication. It was found that even a small amount of communication resulted in a substantial improvement in performance

Table 1: The overshoots and the numbers of communication instants for different values of the threshold T for two interconnected subsystems

T	O_1	O_3	N_1	N_3
0	8.35	4.53	100000	100000
1	8.56	4.80	75	85
3	8.57	4.90	23	13
4	9.20	6.29	19	9
5	13.28	9.20	11	5
10	16.72	21.48	6	4
∞	68.55	57.03	0	0

A summary of the results of the simulation studies are shown in Table 1. The Table shows the overshoots in the outputs for different time-instants N_1 and N_3 at which communication takes place. At instants when a subsystem receives information from the other, it uses that information to generate its control input. At those instant where no information is received, it uses the desired value of that signal in its computations. The Table 1 clearly indicates that substantial improvement in performance that can be achieved by communicating at critical instants (in the present case when the actual value at any subsystem deviates significantly from the desired value).

3.2 Simulation Studies (Six Interconnected Systems)

Since our ultimate aim is to apply the theoretical results in Section 2 to large interconnected systems, simulation studies similar to those described in the previous case were also carried out on a network consisting of six subsystems.

Each subsystem measures its own output y_i, and its error ($y_i - y_{di}$), and broadcasts the values of y_i to all the other subsystems when the absolute value of the error $|y_i - y_{di}|$ exceeds a threshold T_o. In Table 2 the threshold T, the overshoot O, and the number of instants N during which subsystem Σ communicates with the other is shown.

Table 2: The overshoots and the numbers of communication instants corresponding to x_1 for different values of the threshold T for six interconnected subsystems

T	O_1	N_1
0	33.36	100,000
5	40.76	313
10	51.05	108
20	140.83	56
∞	10^{25}	0

The Table shows that there is dramatic improvement in performance even with communication at relatively few instants of time.

4. Conclusions and Future Work

Prior to starting work on this project the PI had theoretically demonstrated that interconnected subsystems can achieve zero tracking errors without any communication provided the value of the desired outputs of all the subsystems are common knowledge. The result was both simple and elegant, and conceptually very appealing. In the spirit of adaptive control, it states that when the values of certain variables are unknown, their desired values can be used instead to compute control inputs. However, the result had been derived only for a simple class of systems. The first part of the project was consequently spent on determining the general classes of systems for which this result is valid.

Simulation studies were then carried out on computer models, to determine how well such control algorithms would perform in applications. It was found that the transient errors of all the subsystems can be substantial with reasonable errors in the parameters and initial condition of the various subsystems. In view of this, the PI decided to use communication between subsystems.

Partial communication between subsystems implies that each subsystem has knowledge of the actual signals needed to compute the proper control inputs at certain instants of time, and has to resort to prior information to generate control inputs at other instants. This gave rise to important stability questions which were resolved in the second stage.

Once the stability issue was decided, attention was focused on the performance of the overall system. To minimize communication costs while achieving a certain level of performance, questions as to when communication between subsystems should take place were raised. Simulation results indicated that significant improvement in performance could be achieved by communicating at a few critical instants of time. At the end of the project optimization problems were being discussed related to when and with whom any subsystem should communicate.

The approach proposed in the report has profound philosophical as well as practical significance for the design of intelligent control systems. In any control system, stability and performance are two objectives that are usually coupled. Stability can generally be assured for restricted classes of systems under fairly strong assumptions. On the other hand, if systems are optimized, it is very hard to determine the conditions under which they will be stable. By proving stability unconditionally at the lower level, the approach used in this report decouples the two problems and permits optimization at a higher level to be carried out independently.

There are numerous directions in which the work reported here can be extended. The most important of these concerns interconnections between subsystems that are nonlinear. If theoretical results can be derived in such cases and implemented using artificial neural networks, it will have a significant impact on the design of large scale control systems.

A number of assumptions have been made throughout the report. For the most part idealized cases have been considered and these assumptions need to be relaxed. Further, it is assumed that communication between subsystems is instantaneous and noise free. The effect of delay in communication is an important question that needs to be addressed. Finally, numerous stochastic versions of the problems described in the report can also be formulated.

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13

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Appendix A

The paper attached is to be published in a Special Issue of the IEE Proceedings on Control Theory and Applications of the United Kingdom in September, 2006.

Decentralized Adaptive Control With Partial Communication

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Abstract

The adaptive control of strictly decentralized adaptive control systems has been investigated in the past and it has been demonstrated that, under very general conditions, exact asymptotic tracking by the different subsystems can be achieved without any communication between them. In this paper the problem is discussed when communication between the different subsystems is permitted. The problems considered range from totally decentralized control to totally centralized control, with emphasis on partial communication. In all cases, the problems can be posed as ones related to the stability of switching systems. The principal theoretical contribution of the paper is the demonstration that the overall system is globally stable, resulting in exact asymptotic tracking by all the subsystems. Decoupling stability and performance issues permits the designer to focus on improving the behavior of the overall system. The principal practical contribution of the paper is the demonstration that significant improvement in transient responses of the subsystems can be achieved with communication at relatively few instants of time.

1 Introduction

After robustness issues related to adaptive control systems were resolved to a large extent in the 1980s, interest in the field shifted to the behavior of interconnected adaptive systems [1]–[3]. A wide spectrum of problems were formulated in the 1990s depending upon the assumptions made about the subsystems, the structure of the interconnections, the nature of the uncertainty, the common knowledge shared by the controllers, and the communication permitted between the subsystems. As in classical adaptive control, convergence of the tracking errors of the subsystems to zero in the ideal case, and robustness of the overall system in the presence of different types of perturbations were investigated. Much of the work reported was devoted to strictly decentralized systems in which no communication between subsystems was permitted. In the Eleventh Yale Workshop on Adaptive and Learning Systems [4] and later in [5], the authors proved that the overall adaptive system could be made globally asymptotically stable with all tracking errors tending to zero, provided all the controllers are aware of the desired trajectories of the other subsystems (*i.e.*, there is implicit cooperation between the different subsystems). The same result had also been obtained earlier independently by Mirkin [6]. During the past three years, the results in [5] were extended to more general classes of systems by Narendra and Oleng in [7],[8]. These include subsystems whose interconnections are nonlinear, whose unknown transfer functions are positive real, and linear and nonlinear subsystems whose outputs but not state vectors are accessible. In all cases it was shown that common knowledge of the objectives of all subsystems is adequate to achieve exact model following.

In [4], the authors stated their belief that communication between subsystems is essential for achieving desired robust performance in complex interconnected adaptive systems, and went on to say that the study of strictly decentralized control systems was merely a precursor to the analysis of such subsystems. In this paper, adaptive control in decentralized systems with partial communication is discussed. Since communication is intermittent, the various controllers in the overall system have to generate their control inputs based either on prior information, or the information received. This gives rise to a class of switching systems whose stability is investigated in this paper. Simulation studies are also included to demonstrate that substantial improvement in performance can be achieved with relatively little communication at judiciously chosen instants of time.

Comment 1: As shown in the paper, while the results for the strictly decentralized case are theoretically very important, nevertheless they are not practically viable. This is because the transient responses of the subsystems in the absence of communication may reach very large values (possibly of the order of 10^{25}) before the errors converge to zero, if the initial parameter errors are large. In contrast to this, transient responses encountered with partial communication are comparable to those generally encountered in adaptive systems. It should also be mentioned that adaptive systems are nonlinear systems, and even after decades of work, efficient mathematical tools do not exist which can predict accurately the transient performance that can be expected. In view of this, one can only rely on simulation results to justify the efficiency of communication for improving transient responses in adaptive control.

The general problem of incorporating communication in control has been addressed from various vantage points by numerous authors [9], [10]. For instance, Hristu and Morgansen [9] consider schemes for stabilizing multiple coupled linear systems with a single controller using output feedback, when the subsystems can only communicate through a network. Walsh *et al* [10] have developed a scheme for networked control systems in which ways are explored for assigning priority for network usage. During the past few years the effective use of communication in control has become an active area of research.

The problem explored in this paper approaches the use of communication in control from a different perspective. Instead of a single controller, each subsystem Σ_i has a controller C_i which, at any instant, generates a control input u_i regardless of whether or not it receives any information from the other subsystems. This results in a switching control system, since the rule for the computation of u_i depends on the information available to C_i at any instant. Also, the decisions concerning what to communicate, and when to communicate it, are left to the individual controllers.

The objective of the study is to determine protocols for controllers to broadcast the values of their outputs so that all the other subsystems use those values to generate appropriate inputs adaptively. For different protocols, the problem is to assure theoretically the stability of the overall system, and study through simulations the improvement in performance in terms of the transient responses of the individual subsystems.

The paper is organized as follows: In Section 2 the Strictly Decentralized Control Problem is described. In Section 2.1 the simplest problem when the interconnections are linear and the state vectors of all the subsystems are available to every controller C_i $(i = 1, 2, \ldots, N)$ is posed, and the proof of stability is presented in Section 2.2. In sections 2.3-2.6 successively more complex versions of the problem are considered in which either the interactions are nonlinear, or less information concerning the other subsystems $\Sigma_{j(j\neq i)}$ is available to each subsystem Σ_i. In all cases, the input u_i and the changes needed to assure global stability are stated. Finally, in Section 3, the decentralized control problem with partial communication is addressed and simulation results are presented in Section 4.

Comment 2: If there is no communication between the subsystems Σ_i, we have the strictly decentralized adaptive control problem treated in [4–6]. If at every instant each subsystem Σ_i broadcasts to all the others all the relevant information needed for control, we have a problem that is equivalent to centralized control. Hence, the problem that is of practical interest is when communication between subsystems is intermittent.

The principal theoretical contribution of the paper is the demonstration that the switching algorithms proposed for all the controllers C_i results in global stability of the overall system. The principal practical contribution of the paper is the demonstration that significant improvement in the transient response of the overall adaptive system can be achieved with communication between subsystems at relatively few instants of time.

Comment 3: Both discrete-time and continuous-time interconnected adaptive systems have been theoretically studied. However, due to space limitations, only theoretical results derived for continuous-time systems are included here. Also, simulation studies have been carried out using both types of systems. Since discrete-time results are simpler to interpret, they are included here.

2 Decentralized Adaptive Control

In this section, we state several different versions of the strictly decentralized control problem. In Section 2.1 the simplest case is considered where all the subsystems, as well as the interconnections, are linear. For the sake of continuity, and also because it features prominently in the following sections while dealing with stability of systems with partial communication, the proof of stability in the above case is included in Section 2.2. In subsections 2.3 to 2.6, more general cases of strictly decentralized control problems are stated. As might be expected, the solutions to these problems become increasingly more complex. Due to space constraints, detailed proofs are given in some cases, and condensed versions of the proofs are presented for the others. In each case, it is shown that the stability of the overall decentralized system can be assured even when there is no communication between the subsystems. This sets the stage for the statement and discussion of the principal problem of decentralized control with partial communication in Section 3.

2.1 The Linear Strictly Decentralized Control Problem

Problem 1: A system Σ consists of N subsystems $\Sigma_1, \Sigma_2, \cdots, \Sigma_N$ that are interconnected. For convenience we shall assume that each subsystem Σ_i has a controller C_i which computes the control input u_i to Σ_i. The subsystems Σ_i are described by the equations

$$\Sigma_i: \quad \dot{x}_i(t) = A_i x_i(t) + b_i \left[\sum_{j=1, j\neq i}^{N} l_{ij}^T x_j(t) + u_i(t) \right] \tag{1}$$

$i \in \{1, 2 \ldots, N\} = \Omega$, $u_i(t) \in R$ is the input, $x_i(t) \in R^{n_i}$ the state of Σ_i at time t, and the constant vectors b_i are known. The matrices $A_i \in R^{n_i \times n_i}$ are assumed to be unknown, but it is further assumed that constant vectors $k_i^* \in R^{n_i}$ exist such that

$$A_{mi} = A_i + b_i k_i^{*T} \qquad \forall i \in \Omega \tag{2}$$

where A_{mi} are asymptotically stable matrices. The terms $l_{ij}^T x_j(t)(j \neq i)$ in equation (1) correspond to the perturbations on the subsystem Σ_i due to subsystems $\Sigma_j(j \neq i, j \in \Omega)$. The vectors $l_{ij} \in R^{n_j}$ are assured to be constant and unknown.

The N subsystems have N reference models Σ_{mi}, where Σ_{mi} is described by the equation

$$\Sigma_{mi}: \quad \dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + b_i r_i(t) \qquad \forall i \in \Omega \tag{3}$$

where $r_i(t)$ are bounded, piecewise-continuous reference inputs and $x_{mi}(t)$ are the corresponding state vectors that have to be tracked.

The strictly decentralized control problem can be stated as follows:
Given N subsystems described by equation(1), and N reference models described by equation (3), and assuming that controller C_i of Σ_i has access only to its own state $x_i(t)$ $(i \in \Omega)$, can it generate an input $u_i(t)$ such that all the signals in the system are bounded, and

$$\lim_{t\to\infty} ||x_i(t) - x_{mi}(t)|| = 0 \tag{4}$$

Comment 4: If the state $x_j(t)$ of Σ_j is accessible to Σ_i, the problem reduces to a standard centralized adaptive control problem. $u_i(t)$ would then include a component $-\hat{l}_{ij}(t)x_j(t)$ to compensate for the perturbations, where $\hat{l}_{ij}(t)$ is an estimate of l_{ij} and is determined on-line using standard adaptive techniques. The principal difficulty arises because both l_{ij} and $x_j(t)$ are unknown to C_i.

2.2 Proof of Stability (decentralized case)

In [5,6] it was shown that if all the subsystems use the (known) desired values of the outputs of the other subsystems in place of their (unknown) true values, the overall system will be stable and all the tracking errors will tend to zero.

The proof given below follows that in [5].

If x_{mj} is used in place of x_j in the generation of the control input $u_i(t)$ of Σ_i, we have terms of the form

$$\sum_{i=1}^{N} b_i(k_i - k_i^*)^T x_i \quad \text{and} \quad \sum_{j=1, j\neq i}^{N} [l_{ij}^T x_j - \hat{l}_{ij}^T x_{mj}] \tag{5}$$

in the error equations, if we choose the control input as

$$u_i(t) = r_i(t) + k_i^T x_i(t) - \gamma_i e_i^T P_i b_i - \sum_{j=1, j\neq i}^{N} \hat{l}_{ij}^T x_{mj} \tag{6}$$

where $e_i = x_i - x_{mi}$ is the state error of Σ_i, P_i is a symmetric positive definite matrix satisfying the Lyapunov equation $A_{mi}^T P_i + P_i A_{mi} = -Q_i$, where Q_i is any symmetric positive definite matrix, and $k_i(t)$ is a vector of adjustable gains.

The differential equation governing the behavior of $e_i(t)$ is given by

$$\dot{e}_i = A_{mi} e_i + b_i \tilde{k}_i^T x_i - \gamma_i e_i^T P_i b_i + b_i \sum_{j=1, j\neq i}^{N} [l_{ij}^T x_j - \hat{l}_{ij}^T x_{mj}] \tag{7}$$

where $\tilde{k}_i = k_i - k_i^*$.

The adaptive laws for adjusting k_i and l_{ij} are chosen as

$$\dot{k}_i = \dot{\tilde{k}}_i = -e_i^T P_i b_i x_i \tag{8}$$

and

$$\dot{\hat{l}}_{ij} = -\dot{\tilde{l}}_{ij} = -e_i^T P_i b_i x_{mj} \quad (j \neq i) \tag{9}$$

where $\tilde{l}_{ij} = l_{ij} - \hat{l}_{ij}$.

Choosing the Lyapunov function candidate

$$V(e_i, \tilde{k}_i, \tilde{l}_{ij}) = \sum_{i=1}^{N} e_i^T P_i e_i + \tilde{k}_i^T \tilde{k}_i + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \tilde{l}_{ij}^T \tilde{l}_{ij} \tag{10}$$

the time derivative of V along any trajectory is

$$\dot{V} = \sum_{i=1}^{N} [-e_i^T Q_i e_i - 2\gamma_i (e_i^T P_i b_i)^2] + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} 2 e_i^T P_i b_i l_{ij}^T e_j \tag{11}$$

or

$$\dot{V} \leq \sum_{i=1}^{N} [-\lambda_{min}(Q_i) \|e_i\|^2 - 2\gamma_i (e_i^T P_i b_i)^2] + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} 2 e_i^T P_i b_i l_{ij}^T e_j \tag{12}$$

It can be shown that if γ_i is sufficiently large, *i.e.*,

$$\gamma_i > \frac{1}{2}(N-1) \max_j \left(\frac{\|l_{ij}\|^2}{\lambda_{min}(Q_j)} \right) \tag{13}$$

where $\lambda_{min}(Q_j)$ is the minimum eigenvalue of Q_j, $\dot{V}$ is negative semi-definite so that all the output and parameter errors are bounded.

Using standard arguments of adaptive control, it can be shown that all the output errors e_i tend to zero asymptotically with time.

Comment 5: The Lyapunov function (10) used to prove stability is quadratic in the output errors e_i as well as the parameter errors $\tilde{k}_i$ and $\tilde{l}_{ij}$. It is also worth pointing out that (since the system is decentralized) it contains only terms of the form $e_i^T P_i e_i$ and does not contain the cross terms $e_i e_j$, *i.e.*, it is block diagonal in the output errors. The critical role played by the feedback control input component $-\gamma_i e_i^T P_i b_i$ to the i^{th} subsystem Σ_i in equation (6) towards overall system stability is also evident from the expression for $\dot{V}$ in equation (12).

As stated in Comment 2, the principal difficulty in controlling the system adaptively arises due to the fact that both l_{ij} and $x_j(t)$ are unknown. To compensate for the disturbing signal $l_{ij}^T x_j$, controller C_i has to use a signal component $-\hat{l}_{ij} x_{mj}$ in the control input $u_i(t)$ to Σ_i, and at the same time modify the adaptive law for adjusting the parameter estimates to

$$\dot{\hat{l}}_{ij} = -e_i^T P_i b_i x_{mj} \tag{14}$$

This ensures that terms of the form $e_i^T P_i b_i \tilde{l}_{ij}^T x_{mj}$ do not appear in $\dot{V}$, which is consequently negative semi-definite.

Comment 6: By making the individual subsystems Σ_i to be sufficiently stable, the overall system is also made stable even in the presence of the interconnections between the subsystems. This is assured by the existence of a block-diagonal Lyapunov function for the latter.

2.3 Extensions of the strictly Decentralized Control Problem

Figure 1 shows the different extensions of the strictly decentralized control problem that have been investigated in [7] and [8]. Such problems fall broadly into two categories: (i) those in which the state vector of the subsystems are accessible, and (ii) those in which only the outputs of the subsystems are accessible. In the first category both linear and nonlinear interconnections have been considered and in the second category subsystems with different relative degrees and having either the feedback linearization form or the normal form have been investigated. Due to space limitations, we address only some of the problems in detail. However, in all cases it has been shown in [7,8] that exact output tracking by all the subsystems can be achieved by using either the desired state variables or desired outputs in place of the actual values in both the control inputs to the subsystems and in the adaptive laws for adjusting the unknown parameters.

Figure 1 goes here.

2.4 Nonlinear Interconnections

Problem 2: The first extension of decentralized controls treated in Problem 1 is to systems where the interconnections are nonlinear. Σ_i in this case is described by

$$\Sigma_i: \quad \dot{x}_i = A_i x_i + b_i k_i^T x_i + b_i \left[\sum_{j=1, j\neq i}^{N} f_{ij}(x_j) + u_i \right]$$

where A_i, b_i, k_i are the same as before, but $f_{ij}(\cdot): R^{n_j} \to R$ are known nonlinear functions of the state x_j of Σ_j ($j \in \Omega$). The state variables $x_j(t)$ are not accessible to the controllers C_i ($i \neq j$), and the objectives are the same as in Problem 1.

The case when the function $f_{ij}(\cdot)$ are globally Lipschitz is a simple one and it has been demonstrated that results identical to those obtained in the linear case can be derived if components of the form - $\sum f_{ij}(x_{mj})$ are used in u_i. When the interconnections are not globally Lipschitz but the functions $f_{ij}(.)$ are assumed to be bounded by a polynomial whose order p is known, the condition may be written as

$$|f_{ij}(x_j) - f_{ij}(x_{mj})| = |f_{ij}(x_{mj} + e_j) - f_{ij}(x_{mj})| \leq \sum_{k=1}^{p} \xi_{ijk} \|e_j\|^k \tag{15}$$

where ξ_{ijk} are constants.

From inequality (15), we can deduce that positive constants $\bar{\xi}_{ik}$ exist such that

$$\sum_{j=1, j\neq i}^{N} \sum_{k=1}^{p} |f_{ij}(x_j) - f_{ij}(x_{mj})|^2 \leq \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{p} \bar{\xi}_{ik} \|e_j\|^{2k}$$

If the input u_i to Σ_i $(i \in \Omega)$ is chosen as
$u_i = r_i + k_i^T x_i - \sum_{j=1, j\neq i}^{N} f_{ij}(x_{mj}) - \gamma_i(e_i^T P_i b_i)[1 + (e_i^T P_i e_i)^{p-1})]$
The resulting error equation of the i^{th} subsystem Σ_i has the form
$\dot{e}_i = A_{mi} e_i + b_i \tilde{k}_i^T x_i + b_i \sum_{j=1, j\neq i}^{N} [f_{ij}(x_j) - f_{ij}(x_{mj})] - \gamma_i b_i e_i^T P_i b_i [1 + (e_i^T P_i e_i)^{p-1}]$

If the parameters are adjusted as

$$\dot{\tilde{k}}_i = -e_i^T P_i b_i x_i \sum_{k=1}^{p} (e_i^T P_i e_i)^{k-1} \tag{16}$$

and a Lyapunov function candidate is chosen as

$$V = \sum_{i=1}^{N} [\tilde{k}_i^T \tilde{k}_i + \sum_{k=1}^{p} (e_i^T P_i e_i)^k] \tag{17}$$

the time derivative along any trajectory can then be shown to satisfy the equation

$$\begin{aligned} \dot{V} = \sum_{i=1}^{N} \sum_{k=1}^{p} [-k(e_i^T P_i e_i)^{k-1} e_i^T Q_i e_i] \quad &+ \quad \sum_{i=1}^{N} \sum_{k=1}^{p} 2k(e_i^T P_i e_i)^{k-1} e_i^T P_i b_i \sum_{j=1}^{N} [f_{ij}(x_j) - f_{ij}(x_{mj})] \\ &+ \quad \sum_{i=1}^{N} \sum_{k=1}^{p} k\gamma_i (e_i P_i b_i)^2 (e_i^T P_i e_i)^{k-1} [1 + (e_i^T P_i e_i)^{p-1}] \end{aligned} \tag{18}$$

If γ_i is chosen as

$$\gamma_i > \max_k [\frac{2(N-1)p\bar{\xi}_{ik}}{[\lambda_{min}(P_i)]^{k-1} \lambda_{min}(Q_i)}] \tag{19}$$

$\dot{V}$ can be shown to be negative semidefinite. This also implies that there exist positive constants η_{ik} such that

$$\dot{V} \leq \sum_{i=1}^{N} \sum_{k=1}^{p} -\eta_{ik} \|e_i\|^{2k} \leq 0 \tag{20}$$

Once again, by previous arguments, it follows that the errors $e_i(t)$ tend to zero asymptotically.

2.5 Subsystems With Relative Degree $n^* = 1$

Problem 3: An interesting extension of the decentralized control problem is to subsystems whose outputs, rather than state variables, affect the performance of other subsystems. In this problem we further assume that the transfer functions of all the subsystems have unity relative degrees and that the corresponding reference models have transfer functions that are strictly positive real (SPR). Σ_i in this case has the form

$$\begin{aligned}\Sigma_i: \quad \dot{x}_i &= A_i x_i + b_i[u_i + \sum_{j=1, j\neq i}^{N} f_{ij}(y_j)] \\ y_i &= h_i^T x_i\end{aligned} \tag{21}$$

where $\{h_i^T, A_i, b_i\}$ are observable and controllable, and $f_{ij}(.)$ are scalar functions of their arguments. The signals to be tracked (*i.e.*, the reference signals) are assumed to be generated by reference models described by

$$\begin{aligned}\Sigma_{mi}: \quad \dot{x}_{mi} &= A_{mi} x_{mi} + b_{mi} r_i \\ y_{mi} &= c_{mi} x_{mi}\end{aligned} \tag{22}$$

with strictly positive real transfer functions

$$W_{mi}(s) = k_{mi} \frac{Z_{mi}(s)}{R_{mi}(s)}.$$

As in standard adaptive control (as well as the problems treated earlier), the input of Σ_i is chosen as

$$u_i = \theta_i^T \omega_i - \sum_{j=1, j\neq i}^{N} f_{ij}(y_{mj}) - \gamma_i g_i(\epsilon_i)$$

where

$$\omega_i^T = [r_i, \nu_i^{(1)}, y_i, \nu_i^{(2)}],$$
$$\theta_i^T = [k_i, \theta_{0i}, \theta_{1i}, \theta_{2i}],$$

and ϵ_i is defined later in equation (25).

The terms $-f_{ij}(y_{mj})$ compensate for the interaction terms $f_{ij}(y_j)$, while $g_i(\epsilon_i)$ is a nonlinear feedback term that depends upon the output error and plays an important role in the stability analysis of the overall system.

The variables $\nu_i^{(1)}$ and $\nu_i^{(2)}$ are defined by the differential equations

$$\begin{aligned}F_i: \quad \dot{\nu}_i^{(1)} &= \Lambda_i \nu_i^{(1)} + l_i \theta_i^T \omega_i - l_i \gamma_i g_i(\epsilon_i) \\ \dot{\nu}_i^{(2)} &= \Lambda_i \nu_i^{(2)} + l_i y_i\end{aligned} \tag{23}$$

where $\Lambda_i \in \mathrm{I\!R}^{(n_i-1)\times(n_i-1)}$ and is stable, and $l_i \in \mathrm{I\!R}^{n_i-1}$ and Λ_i, l_i is controllable. Adjoining $\nu_i^{(1)}$ and $\nu_i^{(2)}$ to x_i we obtain an extended vector $\bar{x}_i$ that is the state vector of the system $\bar{\Sigma}_i$ which includes the system Σ_i and the filters F_i.

$\bar{x}_i = [x_i, \nu_i^{(1)}, \nu_i^{(2)}]$ is described by the differential equation

$$\begin{aligned}\dot{\bar{x}}_i &= A_{mn_i} \bar{x}_i + b_{mn_i}[\phi_i^T \omega_i - \gamma_i g(\epsilon_i) + k_i^* r_i] + \bar{b}_i \sum_{j=1, j\neq i}^{N} [f_{ij}(y_j) - f_{ij}(y_{mj})] \\ y_i &= h_{mn_i}^T \bar{x}_i\end{aligned} \tag{24}$$

where $\bar{b}_i = [b_i, 0, 0]^T$ and $h_{mn_i}^T [sI - A_{mn_i}]^{-1} b_{mn_i}$ is SPR.

Defining $\bar{e}_i = \bar{x}_i - x_{mn_i}$ and $\phi_i = \theta_i - \theta_i^*$, we obtain the error equations

$$\begin{aligned}\dot{\bar{e}}_i &= A_{mn_i} \bar{e}_i + b_{mn_i}[\phi_i^T \omega_i - \gamma_i g_i(\epsilon_i)] + \bar{b}_i \sum_{j=1, j\neq i}^{N} [f_{ij}(y_j) - f_{ij}(y_{mj})] \\ \epsilon_i &= h_{mn_i}^T \bar{e}_i\end{aligned} \tag{25}$$

From equation (25) it is seen that the error equations of the overall system consist of the error equations of the N subsystems together with the nonlinear interconnections between the subsystems. Choosing the Lyapunov function candidate

$$V = \sum_{i=1}^{N}[\bar{e}_i^T P_i \bar{e}_i + \phi_i^T \phi_i]$$

where P_i is the positive definite matrix solution to the Lyapunov equation

$$A_{mn_i}^T P_i + P_i A_{mn_i} = -Q_i \quad Q_i = Q_i^T > 0$$

and adjusting the parameter error vector using the adaptive law

$$\dot{\phi}_i = -\epsilon_i \omega_i$$

the time derivative of V along any trajectory can be expressed as

$$\dot{V} = \sum_{i=1}^{N}[-\bar{e}_i^T Q_i \bar{e}_i - 2\epsilon_i g_i(\epsilon_i)] + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} 2\bar{e}_i^T P_i \bar{b}_i [f_{ij}(y_j) - f_{ij}(y_{mj})] \tag{26}$$

Since $y_j = y_{mj} + \epsilon_j$, $\forall j \in \Omega$, and since y_{mj} is bounded, we have the following relation and inequality:
$|f_{ij}(\epsilon_j + y_{mj}) - j_{ij}(y_{mj})| = 0$ if $\epsilon_j = 0$, $\forall j \in \Omega$

(27)

and
$|f_{ij}(\epsilon_j + y_{mj}) - f_{ij}(y_{mj})| \leq F_{ji}(\epsilon_j)$ with $F_{ji}(0) = 0$

(28)

Following along the same lines as before, it can be shown that there exist $\bar{\gamma}_i$ and $g(\epsilon_i)$ such that

$$2\bar{\gamma}_i \epsilon_i g_i(\epsilon_i) > \frac{(N-1)}{\lambda_{min}(Q_i)} \|P_i \bar{b}_i\|^2 \sum_{j=1, j\neq i}^{N} F_{ji}^2(\epsilon_i)$$

From the above discussion it follows that $\bar{e}_i, \phi_i \in \mathrm{L}^{\infty}$ and $\bar{e}_i \in L^2$, so that it follows by Barbalat's lemma that

$$\lim_{t\to\infty}[y_i - y_{mi}] = 0 \quad \forall i \in \Omega.$$

2.6 Relative Degree $n^* > 1$

The most general strictly decentralized control problem that has been investgated includes subsystems whose relative degrees are greater than one, and where only the desired values of their outputs are known. Two cases have been investigated in this context. In view of space limitations, the details of the derivations are not included here.

Case (i): The subsystem Σ_i is defined by the state equations

$$\Sigma_i : \begin{array}{rcl} \dot{x}_{i1} &=& x_{i2} \\ \dot{x}_{i2} &=& x_{i3} \\ \vdots & & \vdots \\ \dot{x}_{in_i} &=& u_i + \sum f_{ij}(y_j) \\ y_i &=& x_{i1} \end{array}$$

It is assumed that the state variables $x_{i1}, x_{i2}, \ldots, x_{in_i}$ are not accessible, and that only x_{i1} is accessible to controller C_i.

In the approach used for this problem the state x_j of Σ_j is estimated using an observer as $\hat{x}_j$, and the input u_i is chosen as

$$u_i = -\sum_j f_{ij}(y_{mj}) + \sum(\text{a linear combination of } \hat{x}_j, z_{in}, z_{in-1} \text{ and derivatives of } y_{mi})$$

where z_{jk} are appropriately defined auxiliary error variables with z_{i1} as the output error. Using a quadratic Lyapunov function

$$V_{in} = \tilde{x}_i^T P_i \tilde{x}_i + \frac{1}{2} \sum_k z_{ik}^2$$

for each subsystem, it can be shown that $V = \sum_i V_{in}$ is a Lyapunov function of the overall system with $\dot{V} \leq \sum_i -[w_{i1}||\tilde{x}_i^2|| + w_{i2} \sum_k z_{ik}^2] \leq 0$.

Case (ii): Similar results have also been obtained for interconnected systems where each subsystem Σ_i can be expressed in the normal form

$$\Sigma_i : \quad \begin{array}{rcl} \dot{x}_{i1} & = & x_{i2} + a_{i1} \sum_j f_{ij}^1(y_j) \\ \dot{x}_{i2} & = & x_{i3} + a_{i2} \sum_j f_{ij}^2(y_j) \\ \cdots & & \\ \dot{x}_{ip} & = & x_{ip-1} + a_{ip} \sum_j f_{ij}^p(y_j) + b_{i1} u_i \\ \cdots & & \\ \dot{x}_{in} & = & a_{in} \sum_j f_{ij}^n(y_j) + b_{iq} u_i \\ y_1 & = & x_{i1} \end{array}$$

where $a_i = [a_{i1}, a_{i2}, \ldots, a_{in}]^T$ and $b_i = [b_{i1}, b_{i2}, \ldots, b_{iq}]^T$ are unknown, the functions $f_{ij} = [f_{ij}^1, f_{ij}^2, \ldots, f_{ij}^n]^T$ are known, the signs of b_{ij} are known and the zero dynamics of Σ_i is exponentially stable for $i \in \Omega$.

Using standard procedures, the state vector x_i is estimated as $\hat{x}_i$ and the input u_i is chosen using the certainty equivalence principle as a function of the estimates $\hat{a}_i$, $\hat{b}_i$ and a linear combination of $\hat{x}_i$, the auxiliary variables z_{in}, z_{in-1}, and the derivatives of the desired signal y_{mi}. For the above system, a Lyapunov function V that is quadratic in both output errors and parameter errors can be shown to exist. Perhaps more relevant to the point is that if the input and the adaptive law are modified as in Problem 1 depending upon the information available, V is a Lyapunov function and hence assures the stability of the overall system.

3 Adaptive Control with Communication

In Section 2, different versions of the strictly decentralized problem were presented and in each case it was shown that control inputs could be generated based on desired rather than actual values of the output to control the system adaptively. If communication between subsystems is permitted, an extreme case would correspond to Σ_j communicating with Σ_i at every instant the value of $x_j(t)$. In this case, the system is no longer decentralized and the results are identical to those with a centralized controller (except that control to subsystem Σ_i is implemented by controller C_i).

The problem therefore is to determine when any subsystem should communicate with another, and precisely what information it should communicate. We shall assume that in all cases the information communicated by Σ_j to Σ_i at any instant t would transform the problem to one of standard adaptive control (for example $x_j(t)$ in Problem 1). Further, since we are interested in both discrete and continuous-time systems, communication can be at finite instants or over finite intervals.

Comment 7: The decentralized adaptive control problem with partial communication has been studied extensively by the authors and numerous protocols for communication have been generated which will be discussed in future papers. In this paper, it is assumed that all the subsystems Σ_i are aware of the desired outputs of the other subsystems Σ_j $(j \neq i)$, and that communication is primarily used to improve the transient responses of all the subsystems.

3.1 Centralized Control

As stated above, if at every instant, all subsystems are free to communicate their outputs to the other subsystems, the problem ceases to be one of decentralized control. Controller C_i would use a component $-\hat{l}_{ij}^T x_j$ to compensate for the disturbing signal $l_{ij}^T\, x_j$, and adjust the estimate $\hat{l}_{ij}$ adaptively as in standard adaptive control. This would result in the control input

$$u_i(t) \quad = \quad r_i(t) + k_i^T x_i(t) - \gamma_i e_i^T P_i b_i - \sum_{j=1, j\neq i}^{N} \hat{l}_{ij}^T x_j \tag{29}$$

and the adaptation laws

$$\dot{\hat{l}}_{ij} = -\dot{\tilde{l}}_{ij} = -e_i^T P_i b_i x_j \quad (j \neq i) \tag{30}$$

and the quadratic Lyapunov function (10) will assure the stability of the system Σ.

3.2 Adaptive Control with Partial Communication

In practice, due to the cost associated with communication, it is desirable to keep the latter to a minimum, while assuring satisfactory performance. This raises several questions related to adaptive control with partial communication. Assuming that subsystem Σ_i receives information concerning $x_j(t)$ from Σ_j at instant t, and at the same time has knowledge of $x_{mj}(t)$, how should C_i use the information to control the system? How will this affect overall system stability and performance? The answers to these questions will in turn determine when Σ_j should communicate to Σ_i information concerning $x_j(t)$.

3.2.1 A Switching Controller

The controller C_i of Σ_i has at every instant t knowledge of only $x_{mj}(t)$ in the absence of communication, and both $x_j(t)$ and $x_{mj}(t)$ if information is received from Σ_j. It has been shown in the preceding section that using either $x_j(t)$ or $x_{mj}(t)$ and the corresponding adaptive laws for adjusting the parameter estimates $\hat{l}_{ij}(t)$, global stability can be assured. Since C_i has no other recourse but to use $x_{mj}(t)$ when there is no communication, the resulting control in the presence of partial communication can be considered as switching control between two different strategies.

3.2.2 A Common Quadratic Lyapunov Function

In the preceding section it was shown that the quadratic function V in equation (10) is a Lyapunov function for Σ if the feedback control u_i in (6) and adaptive laws (9) are used when only $x_{mj}(t)$ is accessible to C_i. The same function is also a Lyapunov function for Σ if the input $u_i(t)$ given in equation (29) and the adaptive law given in equation (30) are used when $x_j(t)$ is available to C_i $(i \neq j)$. This implies that V given in equation (10) is a Common Quadratic Lyapunov Function (CQLF) for the overall switching system and the system will be stable for arbitrary switching between the two strategies. Further, at those instants where $x_j(t)$ is available, C_i can use any convex combination of $x_j(t)$ and $x_{mj}(t)$ in the control input and the adaptive laws, while retaining stability.

If we examine the proof of stability of all the strictly decentralized control problems treated in Section 2, it is evident that a change in the adaptive laws based on actual or desired values of outputs and a change in the control input is needed to assure that a positive definite function V is indeed a Lyapunov function for the problem. If the actual values, rather than the desired values are used both in the computation of the inputs as well as the adaptive laws, V continues to be a Lyapunov function for the modified system. Hence, all the systems considered in Section 2 are globally stable with partial communication.

The above comments imply that the existence of a CQLF decouples completely the problem of stability from that of performance. Since stability is assured, the designer is free to choose how the information concerning $x_j(t)$ is to be used. Numerous strategies have been proposed to improve performance, but due to space limitations we confine our attention to the specific case of Problem 1. Also, as mentioned earlier, since no analytical methods exist for estimating transient behavior, simulation results are provided to demonstrate improvement in performance with communication.

4 Simulation Results

Simulation studies illustrating the effect of communications on the performance of a decentralized adaptive system have been carried out on both discrete time and continuous time systems. For ease of explanation and clarity of presentation, all simulation experiments presented here deal with discrete-time systems. While there are essential differences between continuous-time and discrete-time adaptive systems, the simulation results described in this section are very similar to those observed in continuous-time case, where communication is not at an instant but over an interval of time.

We consider two interconnected systems in this section. The first contains only two subsystems, both of which are of second order. The second, discussed in Section 4.2, contains six subsystems of second order.

4.1 Two Interconnected Second Order Systems

A decentralized adaptive control system consists of two subsystems Σ_1 and Σ_2. The two subsystems are respectively described by the equations

$$\begin{aligned} &\Sigma_1 : \\ x_1(k+1) &= -0.9x_1(k) + 0.5x_2(k) + \alpha_{13}x_3(k) + 1.0u_1(k) \\ x_2(k+1) &= 0.5x_1(k) - 0.25x_2(k) \end{aligned}$$

and

$$\begin{aligned} &\Sigma_2 : \\ x_3(k+1) &= -0.9x_3(k) + 0.5x_4(k) + \alpha_{31}x_1(k) + 2.0u_2(k) \\ x_4(k+1) &= 0.5x_3(k) - 0.25x_4(k) \end{aligned}$$

It can be seen that the coefficients of the interconnection terms are α_{13} (for Σ_1) and α_{31} (for Σ_2) respectively. Further, it can also be verified that the uncoupled subsystems (*i.e.*, with $\alpha_{13} = \alpha_{31} = 0$) are unstable. All parameters of the two subsystems (including the local subsystem parameters as well as the coefficients of the interconnection terms) are assumed to be unknown.

A series of baseline experiments were first performed for comparison purposes. First, the two systems were completely decoupled by setting α_{13} and α_{31} to be zero. Linear adaptive control (with recursive least squares adaptive laws and a step size of 0.5) was used to control the two subsystems to follow sinusoidal reference trajectories $x_{m1}(k) = 5.0\sin(\frac{2\pi k}{50})$ and $x_{m3}(k) = -2.5\sin(\frac{2\pi k}{25})$. The adaptation was continued for a sufficiently long time (*i.e.*, 100,000 time steps). It was found, as expected that, the errors converged to zero with maximum overshoots $O_1 = 5.12$ and $O_3 = 2.80$ in x_1 and x_3 respectively.

In the second experiment, coupling was assumed to exist only one way from Σ_1 to Σ_2 with $\alpha_{13} = 0$ and $\alpha_{31} = -1.15$. This implies that Σ_1 is unaffected by Σ_2 but the latter is affected by the output of Σ_1. However, no attempts were made to compensate for this coupling and both subsystems were adaptively controlled as if they were decoupled from each other. It was found that the control error e_1 of x_1 still converged to zero with $O_1 = 5.12$ (as expected, since Σ_1 was decoupled). However, since no attempt was made to compensate for the "disturbance term" $l_{31}x_1$, the error e_3 did not converge to zero and the overshoot O_3 was observed to be 12.613. This shows that ignoring the coupling as an uncompensated disturbance results in very poor performance. If, however, Σ_2 compensates for the one-way coupling by incorporating corresponding terms in the identification model, adaptive law, and control law (but uses x_{m1} instead of x_1, i.e., assumes no communication), e_3 did indeed converge to zero and the value of O_3 was observed to be 7.20. This demonstrates that exact tracking is possible in this case even without communication by substituting the desired values instead of the actual ones. If full communication from Σ_1 to Σ_2 was assumed and Σ_2 used the actual values of x_1 (not its desired value x_{m1}) in its control strategy, the control error e_3 as expected converged to zero with $O_3 = 5.32$. Hence, in this case, communication did improve the overshoot. However, the improvement was not very significant, presumably because x_1 was most of the time very close to x_{m1} since Σ_1 is not affected by Σ_2.

In the third set of experiments, two-way coupling was assumed between Σ_1 and Σ_2 with $\alpha_{13} = 1.15$ and $\alpha_{31} = -1.15$. Both subsystems were assumed to employ a selective communication policy where they would communicate their local states to the remote subsystem if the latter deviated from the desired value by more than or equal to a threshold T. It can be easily seen that by varying the value of this threshold T, the amount of communication between the subsystems can be changed. For example, if $T = 0$, the situation corresponds to complete communication at every instant. $T = \infty$ corresponds to the case of no communication. In the intermediate cases, higher values T generally correspond to reduced number of instants of communications. It was assumed that each subsystem would use the actual values of the remote states when such actual values were available (through communication). At all other instants when there was no communication and the actual values of the remote states were not available, they would use the desired values in place of the actual states. Table 1 summarizes the overshoots O_1 and O_3 as well as the number of instants N_1 and N_3 during which communication took place between Σ_1 and Σ_2 during the entire run of 100,000 time steps, for different values of T.

T	O_1	O_3	N_1	N_3
0	8.35	4.53	100000	100000
1	8.56	4.80	75	85
3	8.57	4.90	23	13
4	9.20	6.29	19	9
5	13.28	9.20	11	5
10	16.72	21.48	6	4
∞	68.55	57.03	0	0

Table 1: The overshoots and the numbers of communication instants for different values of the threshold T for two interconnected subsystems

It can be seen from Table 1 that even a small amount of communication between the subsystems improves their performance substantially. For example, a threshold value of 5 results in about twice the overshoot as compared to full communication with only about 0.01% of the communication. At the same time, it results in less than 20% of the overshoot observed with no communication (*i.e.*, when $T = \infty$).

In order to indicate when such communication took place, Figure 2 plots the two control errors (*i.e.*, e_1 and e_3) and the corresponding communication indicator signals. The latter are represented by binary signals which are 0 when no communication occurs and 1 when a communication takes place. It can be seen that the few instants of communication are all clustered during the initial transient phase when the control errors have relatively large magnitudes.

Numerous experiments were conducted for different cases corresponding to stable and unstable decoupled plants as well as stable and unstable interconnections. It was generally observed that the effects of selective communication were more significant when either the decoupled plant or the interconnections were unstable. Similar observations were also made when the control objective was regulation (*i.e.,* following constant desired states) rather than following sinusoidal tracking signals.

4.2 Results with Six Interconnected Second Order Systems

To determine the scalability of the method as well as the results reported in the previous subsection, simulation experiments were also conducted on a larger-scale system consisting of six interconnected second order subsystems.

The six subsystems are described by the equations:

$$\begin{array}{lcl} \Sigma_1 & & \\ x_1(k+1) & = & 0.2x_1(k) + 1.0x_2(k) + 0.5x_3(k) + 0.5x_5(k) + 0.5x_7(k) + 0.5x_9(k) + 0.5x_{11}(k) + 1.0u_1(k) \\ x_2(k+1) & = & 0.2x_1(k) + 0.5x_2(k) \end{array}$$

Σ_2

$$\begin{aligned} x_3(k+1) &= -0.5x_3(k) - 0.5x_4(k) + 0.4x_1(k) + 0.4x_5(k) + 0.4x_7(k) + 0.4x_9(k) + 0.4x_{11}(k) + 2.0u_2(k) \\ x_4(k+1) &= 0.5x_3(k) - 0.25x_4(k) \end{aligned}$$

Σ_3

$$\begin{aligned} x_5(k+1) &= 0.2x_5(k) + 1.0x_6(k) + 0.5x_1(k) + 0.5x_3(k) + 0.5x_7(k) + 0.5x_9(k) + 0.5x_{11}(k) + 1.0u_3(k) \\ x_6(k+1) &= 0.2x_5(k) + 0.5x_6(k) \end{aligned}$$

Σ_4

$$\begin{aligned} x_7(k+1) &= -0.5x_7(k) - 0.5x_8(k) + 0.4x_1(k) + 0.4x_3(k) + 0.4x_5(k) + 0.4x_9(k) + 0.4x_{11}(k) + 2.0u_4(k) \\ x_8(k+1) &= 0.5x_7(k) - 0.25x_8(k) \end{aligned}$$

Σ_5

$$\begin{aligned} x_9(k+1) &= 0.2x_9(k) + 1.0x_{10}(k) + 0.5x_1(k) + 0.5x_3(k) + 0.5x_5(k) + 0.5x_7(k) + 0.5x_{11}(k) + 1.0u_5(k) \\ x_{10}(k+1) &= 0.2x_9(k) + 0.5x_{10}(k) \end{aligned}$$

Σ_6

$$\begin{aligned} x_{11}(k+1) &= -0.5x_{11}(k) - 0.5x_{12}(k) + 0.4x_1(k) + 0.4x_3(k) + 0.4x_5(k) + 0.4x_7(k) + 0.4x_9(k) + 2.0u_6(k) \\ x_{12}(k+1) &= 0.5x_{11}(k) - 0.25x_{12}(k) \end{aligned}$$

As with two interconnected systems, each of the six subsystems in this case was assumed to communicate its state to the other subsystems when the latter deviates from its desired value by more than a threshold T. Each subsystem is assumed to employ an adaptive control scheme using remote states when available and the desired values of the remote states when the actual values are not available. As before $T = 0$ corresponds to complete communication at every instant, and $T = \infty$ corresponds to strictly decentralized control.

Due to space limitations, we present in the following table only the results for one of the states x_1. In particular, we present both O_1 (the maximum overshoot in x_1) and N_1 (the number of instants Σ_1 receives communication from the other subsystems) as function of the threshold T.

T	O_1	N_1
0	33.36	100,000
5	40.76	313
10	51.05	108
20	140.83	56
∞	10^{25}	0

Table 2: The overshoots and the numbers of communication instants corresponding to x_1 for different values of the threshold T for six interconnected subsystems

The desired output of x_1 was $x_{m1}(k) = 5.0\sin(\frac{2\pi k}{50})$ (the other desired outputs are not included to conserve space). It is to be pointed out that in all cases the states in all subsystems converged to their desired values with very small error.

The table given above shows the dramatic improvement in performance with communication even at very few instants. In particular, a threshold of 5 results in an overshoot that is almost the same as that which results when complete communication is permitted, with only about 0.3% of the amount of communication. Each of these two overshoots, in turn, were better than when there was no communication ($T = \infty$). This clearly demonstrates the significant improvement in performance due to communication, even if the latter is done relatively infrequently.

5 Conclusions

Although it has been theoretically proved that interconnected subsystems can achieve zero tracking errors without any communication by using the desired outputs in place of the actual ones, such a control scheme may not be practically viable due to extremely large transient errors before they converge to zero. It has been predicted in previous papers by the authors that communication between subsystems is essential to improve performance and robustness of such decentralized adaptive control schemes. A question consequently arises as to how much communication is needed to have a satisfactory transient error.

It is demonstrated in this paper that since there is a Common Quadratic Lyapunov Function (CQLF) for the two systems resulting from both communication and no communication, stability of the overall scheme is guaranteed for any arbitrary switching between the two. Hence, the decision to communicate can be made on considerations based entirely on control performance and communication costs. It has been demonstrated through simulation studies in the paper that a performance comparable to that obtained with full communication can be realized with communication at only a few instants during the period when the transient error is relatively large.

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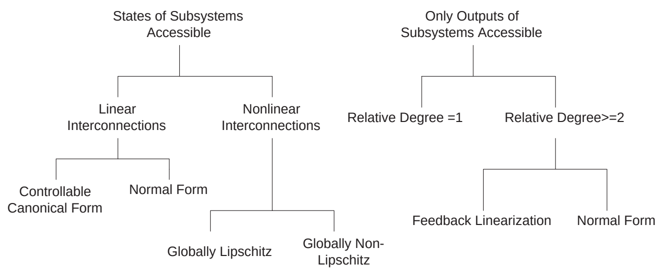

Figure 1: Classes of subsystems considered

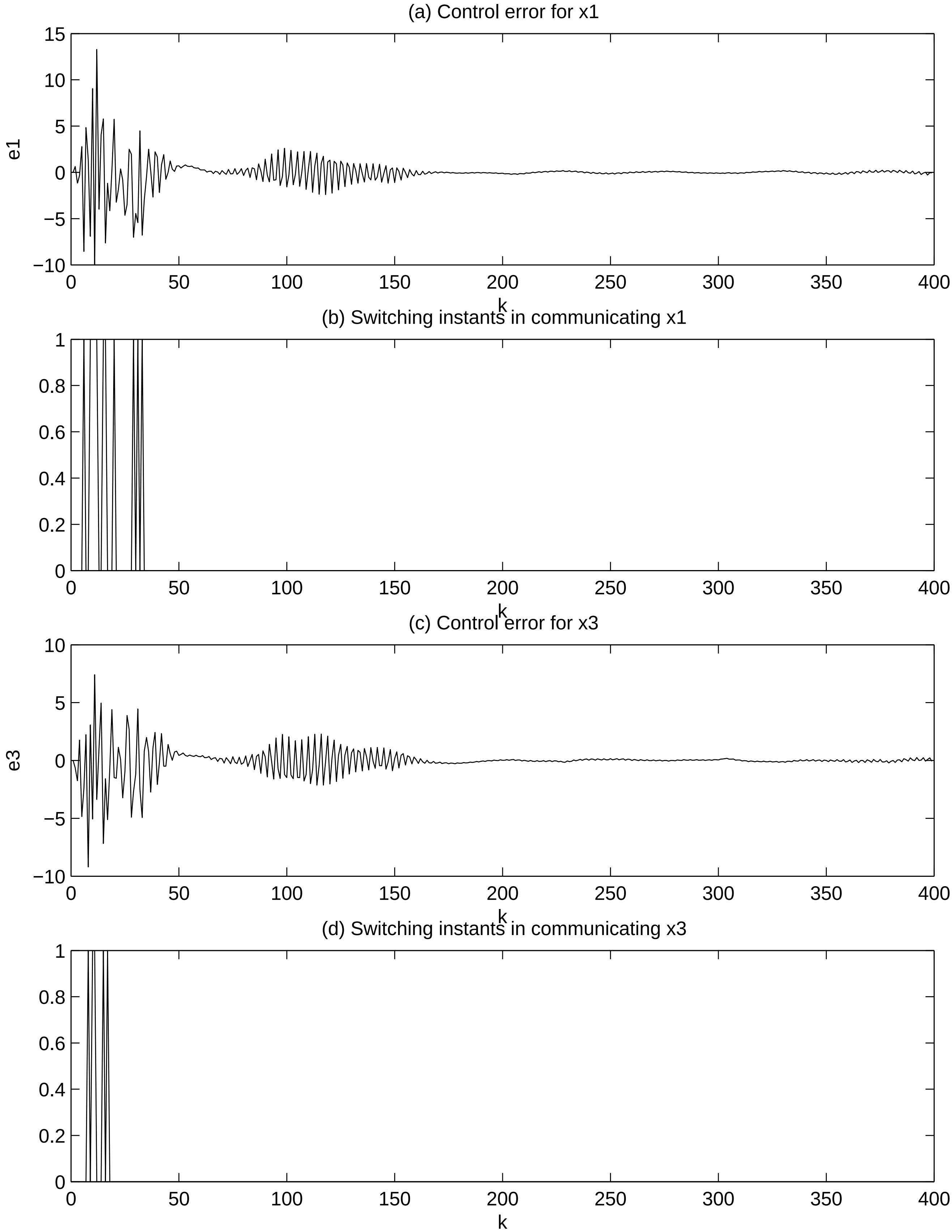

Figure 2: Control errors and communication switching instants for two subsystems example with threshold $T = 5$; (a) and (c): control errors for x_1 and x_3; (b) and (d): switching instants in communicating x_1 and x_3 (with 1 indicating communication and 0 indicating no communication)